Chapter 3: More Deductive Reasoning (Symbolic Logic)

There's no easy way to say this, the material you're about to learn in this chapter can be pretty hard for some students. Other students, on the other hand, absolutely love this stuff. Whichever camp you are in, I suggest taking it slowly. This material is really not all that difficult if you go step by step, trying to understand each idea fully before moving on to the next one.

The good thing is that, from last chapter, you already know the basics of deductive reasoning. This prior knowledge will be useful, considering that the current material is formal deductive reasoning. Sometimes this sort of logic is called *symbolic logic* since we are basically reducing arguments to symbols. It can be called *truth functional logic*, referring to the idea that with deduction itself, the truth of the conclusion is a direct function of the truth (or lack thereof) of the premises. However, we will use the term symbolic logic.

Ultimately you are going to be turning deductive arguments into symbols and then manipulating those symbols to show that the arguments are valid. This part of logic and critical thinking is where math and language come together, so those of you who like clear answers are going to like this material. There is no ambiguity here, either you get the correct answer or you don't, just like with math. In fact, there are direct similarities between math and symbolic logic. For example, when we show that arguments are valid we call them *proofs*—proofs are also done in math classes (to prove a theorem, for example).

Symbolic Reasoning

We all know what symbols are, don't we? To oversimplify the definition, symbols are things that stand for other things. In our culture, with traffic lights, the color red symbolizes *stop* and the color green symbolizes *go*. Symbols make the world easier for us to navigate. We don’t have to write out “stop” and “go” because we have collectively agreed to represent those actions with colors.

Something similar happens with symbolic logic. Symbolic deductive reasoning is used when other forms of reasoning would be too slow. Let's take validity, for example. It’s easy to see that simple arguments are valid or invalid. Here are a couple of examples:

This argument is obviously valid:

1. All men love hot dogs.
2. Steve is a man.

So Steve loves hot dogs.

If it’s true that all men love hot dogs (remember, we only *assume* truth with validity) then it has to be true that Steve loves hot dogs if Steve is a man. Clearly this is a valid argument that is easy to assess—if the premises are assumed to be true, then the conclusion follows with certainty.

This argument is obviously invalid:

1. Some women love hot dogs.
2. Joanne is a woman.

So Joanne loves hot dogs.

If only some women love hot dogs, it does not follow that, just because she is a woman, Joanne loves hot dogs. If only some women love hot dogs, then Joanne might be one of the women who does not like them. So the argument is obviously invalid since the conclusion can be false, even when we assume the premises are true.

Now, the above are simple arguments that are easy to assess. But what about more complicated arguments with long, complex premises? Consider a religious debate between two people. One, call him Tim, argues that there is a God and that sinners will suffer eternal damnation. The other, call her Julie, argues that, if God is all good, forgiving, and compassionate, then there can be no hell of eternal suffering when we die. Here is the way Julie's argument looks when put it in premise/conclusion format:

> 1. If God does not exist, then there will be neither a heaven nor a hell for us when we die.
> 2. If he does exist, then there should be human suffering only if this suffering contributes to fulfilling God's purpose.
> 3. However, if there is to be human suffering and eternal suffering, then this cannot contribute to fulfilling God's purpose (because God is supposed to be good, forgiving, and compassionate).
> 4. There will be human suffering and eternal suffering, if there is a hell for us when we die.
>
> It follows that there will not be a hell for us when we die.

Now, given your skills in determining validity, you could probably take some time to determine whether or not the above is valid or invalid. But it would take a while, and it might not be a very fun thing to do. Luckily for us, there is an easier way—this is where symbolic logic comes in. Similar to mathematics, symbolic logic was invented so we can follow long trails of reasoning that are not easy to otherwise assess. Sometimes logic or reasoning in general is defined as "systematic common sense." This definition applies especially to symbolic logic, which puts arguments and common sense into a system, as we'll see.

Think about math. When you are solving an equation, however simple, do you write out the numbers? Is it "75 X 6" or "Seventy five times six?" Clearly, the first way is the way we do math. Equations would be incredibly difficult to figure out if we didn't have symbols for numbers and functions. In math, we systematize numbers and their relations; in symbolic logic we systematize language.

This being said, just like math, symbolic logic can get incredibly complicated and full classes are devoted to it in upper division philosophy departments. The history of symbolic logic actually goes all the way back to Aristotle, who was the first well-known Western philosopher to turn arguments into symbols over two thousand years ago in ancient Greece.[1] Many other Roman, Christian, and Islamic thinkers helped to develop symbolic logic over the years. But it wasn't until the late 19th century/early 20th century that symbolic logic had its renaissance. Important founding figures here are George Boole, Gottlob Frege, Bertrand Russell, and more.[2] If we go later into the 20th century, we also find figures like Kurt Gödel and Alfred Tarski.[3]

Since this is an *introduction* to logic class, we'll just scratch the surface here. Nevertheless, I hope you will see the importance of symbolic logic to our present age, particularly with respect to computers. The contributions that the founding figures mentioned above made to symbolic logic were a major factor in the computer revolution, and ultimately our development of the sophisticated technology we have today, as we will see.

Symbolic Translation

The beginning of symbolic logic is learning to translate ordinary, natural language, like what I'm writing now, into more basic

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[1] Aristotle. (2012). *The Organon*. R. B. Jones (Ed.). CreateSpace.

[2] See the following: Boole, G. (2016). *The Mathematical Analysis of Logic*. CreateSpace; Frege, G. (1980). *The Foundations of Arithmetic*. Evanston, Il: Northwestern University Press; Russell, B. (2010). *Mysticism and Logic: And Other Essays*. CreateSpace.

[3] Gödel, K. (1992). *On Formally Undecided Propositions of Principia Mathematica and Related Systems*. Cambridge, UK: Cambridge University Press; Tarski, A. (1956). *Logic, Semantics, Mathematics*. Oxford, UK: Clarendon Press.

symbols.[4] First of all, this little symbol "/.:" means "therefore" and refers to the conclusion of an argument. That will be important to remember for this material, but as you'll see below, there is much more to convert to symbols.

Completely understanding symbolic logic is like learning a new language (in fact, some philosophy graduate programs will consider proficiency in symbolic logic suitable for a foreign language requirement).

The beginning stage is often referred to as *propositional logic* because it is concerned with trying to understand the connections between propositions (or claims/statements) in ordinary, natural language. Consider the following: "Either John passes the final or he will not pass the course."

There are two statements here that are linked: "John passes the final" and "John passes the course." They are linked by "or" and "not." These linking terms are called "logical connectives."

Logical Connectives: and, or, not, if/then, if and only if.

Then there are variables. In symbolic logic, we use variables to stand for statements. For example, we use "F" to stand for "John passes the *final* exam." Notice that we could have used "J" instead. There is no exact science to choosing a variable—the idea is to choose a term from the statement that seems to represent that statement best.

So, now we are seeing the components of the symbolic language we'll be using: variables and logical connectives. There are symbols that are used to stand for logical connectives, they are below. No matter how complicated this stuff gets, just remember

[4] Language after all, is in some sense a representation/symbol for the literal sound of speaking.

that all we're dealing with (at least in this class) boils down to variables and logical connectives, that's it.

> *Variables*: A, B, C...
>
> *Symbols for Logical Connectives*:
> And: &
> Or: v
> Not: ~
> If/then: →
> If and only if: ↔

Logicians (people who study symbolic logic) give labels to statements with each of these connectives:

> *Conjunctions* are "and" statements (&).
> *Disjunctions* are "or" statements (v).
> *Negations* are "not" statements (~).
> *Conditionals* are "if/then" statements (→).
> *Biconditionals* are "if and only if" statements (↔).

Although a conditional is a single symbol, it often contains two terms in ordinary language: "if" and "then." This can be confusing, but just remember the nature of the claim is that one thing depends on the other. "If we eat, then we'll be full." In conditionals, what comes first is called the "antecedent" and what follows is called the "consequent." However, sometimes I will also refer to the first term in other types of claims as the antecedent and the second term as the consequent.

Given the symbols above, how would we translate the following into symbolic notation? "Juan went to the store and Mary stayed home." The first step is to choose variables. Let's choose "S" for "Juan went to the *store*" and "H" for "Mary stayed *home*." Now, the final symbolic notation would be: "S & H." I hope you're able to see why this is the case. Here are a few more examples:

If I eat fish, then you'll eat pork: F → P (this is a conditional).

We'll go to the mall if and only if you get out of bed: M ↔ B (this is a biconditional).

Either we get a house or an apartment: H v A (this is a disjunction).

We're not going to the beach: ~ B (this is a negation).

We are going to the beach: B (this is an affirmation that B took place/happened)

The essential point is to see that language can be put into a system of symbols, and this is exactly what symbolic logic does. But this stuff can get complicated quickly. Here's an example of what a full argument of symbolic logic looks like.

1. (A & B) → [A → (D & E)]
2. (A & B) & C /.: D v E
3. A & B
4. A→ (D & E)
5. A
6. D & E
7. D
/.: D v E

It's important to note that we could replace the variables and logical connectives in this entire argument, and we would have an argument in natural language. Just to be completely clear, when I refer to natural language, I simply mean the ordinary, everyday language that you read and speak on a daily basis.

What about the fact that people speak different languages? Does that change the logic? Do different languages have different logic? They might. Though it's only anecdotal evidence, or nongeneralizable evidence from my own experience (see chapters 4 and 7), I have a few friends who claim that it is more difficult to represent Western logical arguments in Mandarin Chinese. I don't speak Spanish very well, though I've written and spoken it a little, but it's a Western language, so the logic doesn't seem to me to differ too much from English, aside from things like word order, adjective placement, and the like.

How is language related to logic and reasoning? It's a deep question, one that many brilliant thinkers have pondered, and continue to ponder.[5]

Computers: Logic Machines

Websites are constructed using HyperText Markup Language (HTML). If you've built webpages, or worked with more complicated programming languages (like C++), you'll be quite familiar with all of this. Like these programming languages, and giving rise to them, symbolic logic is a language of symbols with different names and functions.

It is languages like these that underlie the computer revolution and the general functioning of computers. Like most human achievements, the computer revolution has many different fields of study to thank, but the development of deductive, symbolic languages played a huge role. In this sense, as noted in the last chapter, *computers are logic machines*, carrying out operations to their logical conclusions, dictated by the rules of the programming language.

Proofs of Validity

But now, back to the central concerns of this chapter. To reiterate, symbolic logic was originally used to show the validity or

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[5] For example see the works of Noam Chomsky, Richard Rorty, Susan Haack, and Ludwig Wittgenstein on the topic.

invalidity of arguments that are long and complex. We've learned how to translate from ordinary language to symbolic logic, and the next step is to test whether arguments in this symbolic language are valid—these are usually called *proofs of validity.* There are different ways to prove validity, but we'll be focusing on one. In the chapter 2 homework, you had to determine whether shorter arguments are valid. Again, keep in mind that all we're doing with this current material is proving validity with longer arguments that have been turned into symbols.

There are other methods of proving validity (as well as invalidity) of long arguments. For example, one common method that we won't be covering in this class is the truth table method.[6] In this class we will be focusing on the method with which I have the most expertise. The method we're using is sometimes called *deduction*. We will be using deductive reasoning to construct proofs of validity. Basically, we are going to be taking argument patterns or forms that are already valid and using them to derive a conclusion from a group of premises. It sounds more difficult than it is. However, I do suggest that you read carefully from this point forward, and make sure that you've understood each section before moving on to the next.

The 9 Valid Argument Forms

We'll be working with 9 valid argument forms, all explained in detail below. After the explanations, the argument forms are listed on a single page; when you begin to solve the proofs in the homework, you will be heavily referencing that page.

When discussing each form below, I use natural language arguments to show why each is valid, then I show the argument in symbolic form. Hopefully the natural language will help you see

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[6] See the Wikipedia article on it for an overview if you're interested: Truth table method. (n. d.). In *Wikipedia*. Retrieved July, 17, 2017, from https://en.wikipedia.org/wiki/Truth_table.

that the argument forms are not arbitrary. However, once you start constructing proofs, there will only be symbols involved.

Modus Ponens

Here is the first argument form in natural language:

1. If we ever see a gas station, we always stop.
2. We saw a gas station.

Therefore, we stopped.

Now, let's translate it. First, our variables. We'll let "seeing a gas station" be represented by "G" and "stopping" be represented by "S." Now, how does the argument look in symbolic form given our variables? Something like this:

1. $G \rightarrow S$
2. G

/.: S

This is one of the most basic valid argument forms: modus ponens. Notice that if the two premises are assumed to be true, then the conclusion must also be true—so it is indeed valid.

Just to be absolutely clear, we are going to be using this and the other argument forms discussed below (for a total of 9) to show that a conclusion can be derived/proved from a set of premises.

Modus Tollens

Here is the 2nd argument form in natural language:

1. If I am Socrates, then I invented the Socratic Method.
2. I did not invent the Socratic Method.

So I am not Socrates.

Let “me being Socrates” be represented by “S” and “me inventing the Socratic Method” be represented by “M.” Now, let’s see what the argument looks like in symbolic form:

1. $S \rightarrow M$
2. $\sim M$

/.: $\sim S$

If I didn’t invent the Socratic Method, then I can’t be Socrates. The argument is valid since the conclusion follows with certainty assuming the premises are true.

Chain Argument

Here is the 3^{rd} argument form in natural language:

1. If I go to school, I’ll get a job.
2. If I get a job, I’ll get money.

So if I go to school, I’ll get money.

Let “me going to school” be represented by “G,” let me getting a job be represented by “J,” and let “me getting money” be represented by “M.” So in symbolic form, the argument looks like this:

1. $G \rightarrow J$
2. $J \rightarrow M$

/.: $G \rightarrow M$

Perhaps this form in particular shows the usefulness of symbolic logic best: when you look at this argument in symbolic form, it seems easier to see the link between premises and conclusion, and the central role the middle term “J” plays.

Again, notice that if the premises are assumed to be true, then the conclusion follows with certainty. For the last 6 rules, I will refrain from mentioning validity in all cases, and I will assume that the validity of the arguments is understood.

Disjunctive Argument

Here is the 4th argument form in natural language:

1. I will eat or I will sleep.
2. I did not eat.
Thus, I slept.

Let "me eating" be represented by "E" and "me sleeping" be represented by "S." Here is the argument in symbolic form:

1. E v S
2. ~ E
/.: S

Notice that the following would be equally valid:

1. E v S
2. ~ S
/.: E

In contrast to a conditional claim, with a disjunction the antecedent and consequent are interchangeable. To understand this clearly, just see that when we turn the immediately prior argument into natural language, it is still valid:

1. I will eat or I will sleep.
2. I did not sleep.
Thus, I ate.

Conjunction

Here is the 5th argument form in natural language:

1. It's raining.

2. It’s cold.
Therefore, it’s raining and it’s cold.

Let “it’s raining” be represented by “R” and “it’s cold” be represented by “C.” In symbolic form the argument looks like this:

1. R
2. C
/.: R & C

If two things are both true separately—in this case that it’s raining and that it’s cold—then they are equally true when we state them together, which is what makes this argument valid. Similar to a disjunctive argument, and unlike a conditional, in the conclusion of a conjunction the antecedent or consequent are interchangeable. So the following would also be valid:

1. R
2. C
/.: C & R

Simplification

Here is the 6th argument form in natural language:

1. I am tired and hungry.
Therefore, I am tired.

Let “me being tired” be represented by “T” and let “me being hungry” be represented by “H.” In symbolic form the argument looks like this:

1. T & H
/.: T

If it’s true that I am both tired and hungry, then we can derive with certainty either that you are tired *or* hungry. So the argument is valid. We can simplify the argument down to the antecedent or consequent, hence the name of this argument form.

Again, similar to a disjunctive argument, and unlike a conditional, the simplification can take either the antecedent or consequent in the conclusion. For example, this is still valid:

1. T & H
/.: H

Addition

Here is the 7th argument form in natural language:

1. I will eat.
So I will eat or sleep.

Or…

So I will eat or do homework.

Yes, you see that right. There are two possible conclusions, and in fact there are an infinite amount of possible conclusions. This is because a disjunctive claim can be true even if the antecedent or consequent is false, so we can join any claim to a conjunction that is a part of an argument and the argument can still be valid.

Let "me sleeping" be represented by "S," let "me eating" be represented by "E," and let "me doing homework" be represented by "H." Here is the argument in symbolic form:

1. E
/.: E v S

Or…

/.: E v H

Again, remember that we could join any claim we want. Instead of "S" or "H" we could add, for example, "G" to stand for "me believing in God."

Constructive Dilemma

Here is the 8th argument form in natural language:

1. If there is food, I will eat.
2. If there is a bed, I will sleep.
3. There is food or there is a bed.

So I will eat or sleep.

Let "there being food" be represented by "F," let "me eating" be represented by "E," let "there being a bed" be represented by "B," and let "me sleeping" be represented by "S." So in symbolic form the argument looks like this:

1. F → E
2. B → S
3. F v B

/.: E v S

This and the next are the most complicated of the argument forms, so it may take some practice to recognize them.

Destructive Dilemma

Here is the 9th argument form in natural language:

1. If there is food, I will eat.
2. If there is a bed, I will sleep.
3. I did not eat or I did not sleep.

So there was not food or there was not a bed.

The variables here are the same as with the constructive dilemma, so refer back to the last argument form. Here is what the destructive dilemma given above looks like in symbolic form:

1. $F \rightarrow E$
2. $B \rightarrow S$
3. $\sim E \vee \sim S$

/.: $\sim F \vee \sim B$

Before moving onto recognizing the argument forms in operation and, ultimately, solving proofs, let's look at two prominent deductive fallacies.

Deductive Fallacies

Recall that fallacies are mistakes in reasoning. Chapter 6 focuses on relevance fallacies, or fallacies in which the premises are not relevant to the conclusion of an attempted argument. Chapter 7 focuses on inductive fallacies, in which the premises given are not adequate or sufficient to support the conclusion.

However the deductive fallacies we are studying in this chapter are a bit different, sometimes called *formal fallacies*. Basically, the fallacies in the last two chapters of this reader focus on the content within an attempted argument (or the soundness), while the deductive fallacies in this chapter focus on the form of an attempted argument (or the validity). So deductive fallacies are common types of invalid arguments—the opposite of the 9 argument forms, which are common types of valid arguments.

Affirming the Consequent

This is basically a bad modus ponens argument. Whereas modus ponens rightly affirms the antecedent, this one fallaciously affirms the consequent. Here is the form of this fallacy:

1. $P \rightarrow F$
2. F
/.: P

Kind of looks like modus ponens, right? Wrong. Modus ponens has P in the second premise, but this one has F in that spot. It is an invalid argument, or a fallacy. Consider the attempted argument in natural language with content:

1. If Juanita is pregnant, then she is a female.
2. Juanita is a female.
So she is pregnant.

Now it's easier to see why this is invalid. Clearly the premises do not prove the conclusion, since according to the argument's logic, Juanita could be one of the many females who is not pregnant. Notice, however, that we could turn it into a valid argument by switching the first premise to this: "If Juanita is a female, then she is pregnant." Although this change would make the argument unsound, the argument would still be valid and, therefore, no longer the affirming the consequent fallacy (again, these fallacies are related to validity, not soundness).

Denying the Antecedent

Now we turn to what is basically a bad modus tollens argument. Whereas modus tollens rightly denies the consequent, this fallacy fallaciously denies the antecedent. Here is the form:

1. $M \rightarrow P$
2. $\sim M$
/.: $\sim P$

Kind of looks like modus tollens, right? Again, wrong. Modus tollens has ~P in the second premise, whereas this one has ~M in that spot. Consider the attempted argument/fallacy in natural language:

> 1. If I am a man, then I have watched pornography.
> 2. I am not a man.
>
> Thus I have not watched pornography.

Do men watch more pornography than women? Probably. But that doesn't mean that women don't watch it too. In particular, according to the logic of the attempted argument, the speaker could be a woman who watches pornography, so the conclusion is not proven, making the argument invalid.

Recognizing Argument Forms in Operation

With invalid arguments out of the way, the next step is learning to recognize some of the nine valid argument forms in operation. Remember fast and slow thinking? At first, recognizing the argument forms will require slow thinking. But over time, hopefully, you'll be able to train your mind to recognize the forms in different contexts. To make it easier, below is Table 1 that lists all 9 forms with symbols. Remember that the symbols are arbitrary, just like variables in math.

Table 1. The 9 Valid Argument Forms

Modus Ponens (MP) 1. P → Q 2. P /.: Q	Modus Tollens (MT) 1. P → Q 2. ~ Q /.: ~ P	Chain Argument (CA) 1. P → Q 2. Q → R /.: P → R
Disjunctive Argument (DA) 1. P v Q 1. P v Q 2. ~ P 2. ~ Q /.: Q /.: P	Simplification (SIMP) 1. P & Q 1. P & Q /.: P /.: Q	Conjunction (CONJ) 1. P 2. Q /.: P & Q
Addition (ADD) 1. P 1. Q /.: P v Q /.: P v Q	Constructive Dilemma (CD) 1. P → Q 2. R → S 3. P v R /.: Q v S	Destructive Dilemma (DD) 1. P → Q 2. R → S 3. ~ Q v ~ S /.: ~ P v ~ R

Before trying the homework, there is one more important point to understand about this material: anything in parenthesis can stand for a single term. For example, both of the following are modus ponens arguments:

1. $P \rightarrow Q$
2. P

/.: Q

1. $(X \,\&\, \sim Y) \rightarrow Q$
2. $X \,\&\, \sim Y$

/.: Q

I suggest reflecting more deeply on why the above two arguments are both modus ponens—if you don't understand why, the proofs will be that much more difficult.

Now, let's consider the following examples. What argument form was used to reach the conclusion given?

1. $(A \rightarrow \sim B) \,\&\, (\sim C \rightarrow D)$ /.: $A \rightarrow \sim B$

The conclusion, which is $A \rightarrow \sim B$, was reached by the 5th argument form: simplification. So to properly indicate your use of the form (sometimes known as a "rule") as a line in a proof, you would write "SIMP" next to the line it came from. Like this:

1. $(A \rightarrow \sim B) \,\&\, (\sim C \rightarrow D)$ /.: $A \rightarrow \sim B$
2. $A \rightarrow \sim B$ 1, SIMP

Notice that the abbreviations for all the argument forms are in Table 1 above. Let's try another:

1. $(V \rightarrow W) \text{ v } (X \rightarrow Y)$
2. $\sim(V \rightarrow W)$ /.: $X \rightarrow Y$

What rule was used to get the conclusion, $X \rightarrow Y$? It was rule 4, the disjuntive argument.

1. $(V \rightarrow W) \vee (X \rightarrow Y)$
2. $\sim(V \rightarrow W)$ /.: $X \rightarrow Y$
3. $X \rightarrow Y$ 1, 2 DA

This should all get clearer as you try this chapter's homework. The idea is to get comfortable with these nine rules and then move on to the rest of the homework where you'll have to construct proofs of validity yourself. With proofs, you won't just be recognizing these rules, you'll be deciding for yourself which ones to use to derive the conclusion.

For reference, there is a full solution to a proof below. *But*, I recommend that you do not look at it until after you have at least tried the homework. At this point, it may just confuse you. After you have tried some of the easier proofs, it may be helpful to come back to this more difficult proof for reference.

Step-by-Step Solution to a Proof

Here is the proof:

1. $H \rightarrow (I \rightarrow J)$
2. $K \rightarrow (I \rightarrow J)$
3. $(\sim H \& \sim K) \rightarrow (\sim L \vee \sim M)$
4. $(\sim L \rightarrow \sim N) \& (\sim M \rightarrow \sim O)$
5. $(P \rightarrow N) \& (Q \rightarrow O)$
6. $\sim(I \rightarrow J)$ /.: $\sim P \vee \sim Q$

Here are the terms for lines 7 and 8:

7. $\sim H$ 1, 6, MT
8. $\sim K$ 2, 6, MT

Let's stop for a minute before going onto the next lines. How did I get ~H on line 7? Notice that I used modus tollens (MT) from lines 1 and 6, which is written next to ~H on line 7. So let's look at lines 1 and 6:

1. H → (I → J)
6. ~(I → J)

And what is the form of modus tollens? Using generic variables, here's the form:

A → B
~B
/.: ~A

Do you see how the form fits lines 1 and 6? Don't forget that something in parenthesis can be a single term. Let's put them next to each other:

A → B	1. H → (I → J)
~B	6. ~(I → J)
/.: ~A	7. ~H

Notice that the conclusion, ~H, is what we put on line 7, the line we were working with. Getting the ~K is the same, except with line 2 instead of line 1.

Now let's look at the next few lines of the proof/deduction:

9. ~H & ~K	7, 8, CONJ
10. ~L v ~M	3, 9, MP
11. ~L → ~N	4, SIMP
12. ~M → ~O	4, SIMP

Notice that the ~H and ~K come from the lines we just got, lines 7 and 8. And all we do is bring them together, hence a conjunction (CONJ). But notice that a conjunction only uses the &, so you cannot use another connective, like the → or the v. It's important to pay attention to the connectives each time you use a rule.

I just gave you an example using modus tollens (MT), and it is quite similar to modus ponens (MP), so you should be able to figure out why line 10 is what it is. Just match up the form of MP to the lines involved (lines 3 and 9), just as I did above for the ~H on line 7.

Lines 11 and 12 both come from simplifications (SIMP), which is the opposite of a conjunction. You will see that the terms on line 4 are connected by an &. Because of this, you can take either term and put it on a new line; this is what has been done on lines 11 and 12 above. Again, remember that anything within parenthesis can be used as a single term.

The next line of this proof uses a complicated rule, so I am going to focus on this line for a bit:

13. ~N v ~O	10, 11, 12, CD

Here is the generic form of the constructive dilemma (CD) rule next to the lines used to derive line 13:

A → B	11. ~L → ~N
C → D	12. ~M → ~O
A v C	10. ~L v ~M
/.: B v D	13. ~N v ~O

This is tricky, because negatives (~) are being used, but the form remains the same. Also, this illustrates the way you may have to look at the lines slightly out of order occasionally to see the form that is necessary.

Here are the last lines of the proof/deduction:

14. P → N	5, SIMP
15. Q → O	5, SIMP
16. ~P v ~Q	13, 14, 15, DD

Lines 14 and 15 should be clear at this point, I already went over simplification above. And line 16 is a destructive dilemma (DD), rather than a constructive dilemma (CD). These last two forms are similar, so I'm hoping that if you've followed me this far, you'll be able to see *why* line 16 is what it is.

Keep in mind that there may be slightly different ways to solve this proof, though many of the steps will be similar. Also, the location of the lines is arbitrary in some cases—for example I could have reversed lines 14 and 15.

Here is the full solution to this proof with everything together:

1. H → (I → J)	
2. K → (I → J)	
3. (~H & ~K) → (~L v ~M)	
4. (~L → ~N) & (~M → ~O)	
5. (P → N) & (Q → O)	
6. ~(I → J) /.: ~P v ~Q	
7. ~ H	1, 6, MT
8. ~ K	2, 6, MT
9. ~H & ~K	7, 8, CONJ
10. ~L v ~M	3, 9, MP
11. ~L → ~N	4, SIMP
12. ~M → ~O	4, SIMP
13. ~N v ~O	10, 11, 12, CD

14. $P \rightarrow N$	5, SIMP
15. $Q \rightarrow O$	5, SIMP
16. $\sim P \vee \sim Q$	13, 14, 15, DD

Major Ideas for More Deductive Reasoning (Symbolic Logic)

Although anything from the readings or homework might appear on the assessments, the following **major ideas** should be clearly understood.

- Validity (review)
- Proofs of validity
- The 9 valid argument forms
- Deductive Fallacies
- Recognizing the argument forms in operation